Stat 134: Section 13

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Conceptual Review

1. What is moment generating function $M_X(t)$ of a random variable X, what is uniqueness property of mgf?
2. Express moment generating function of $Y = aX + b$ in terms of $M_X(t)$.
3. Express moment generating function of $Z = X + Y$ in terms of $M_X(t)$ and $M_Y(t)$ when X and Y are independent.

Problem 1

1. Find mgf of $Bernoulli(p)$ and $Geom(p)$.
2. Use part 1. to find mgf of $Binomial(n, p)$ or $Negbin(r, p)$
3. Use part 2. to prove that if $X_1 \sim Negbin(r_1, p)$ and $X_2 \sim Negbin(r_2, p)$ and X_1, X_2 are independent, then $X_1 + X_2 \sim Negbin(r_1 + r_2, p)$.

Problem 2

Define $K_X(t) := \log M_X(t) = \log E[e^{tX}]$.

1. Prove $K'_X(0) = E[X]$ and $K''_X(0) = Var[X]$.
2. Now use the fact that $X \sim Gamma(\alpha, \lambda)$ then $M_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^{\alpha}$ to calculate E[X] and Var[X]. Then compare with the result we know.

Problem 3

Derive $M_X(t)$ and $K_X(t)$ when $X \sim Poisson(\lambda)$. Then calculate $E[X]$ and $Var[X]$.

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