Stat 134: Section 15

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Conceptual Review

a. Let X, Y have joint density $f_{X,Y}$. Draw a picture showing what we mean by $P(X \in dx, Y \in dy)$. How do we calculate $P((X, Y) \in A)$ or $\mathbb{E}[g(X, Y)]$?

b. How do we find marginal density of X, Y with joint density $f_{X,Y}$, what happens when X and Y are independent?

c. If X, Y are jointly uniformly distributed over a region, what does their joint density look like?

Problem 1

A metal rod is ℓ inches long. Measurements made using this rod are distributed uniformly from $\ell - 0.1$ to $\ell + 0.1$ inches, accounting for random error. Assume measurements are independent of each other.

a. Find the chance that a measurement is within 0.01 inches of ℓ.

b. Find the chance that two measurements are within 0.01 inches of each other.

Draw a picture to help visualize this event.

Ex 5.1.2 in Pitman's Probability

Problem 2

Suppose that (X, Y) is uniformly distributed over the region $\{(x, y) : 0 < |y| < x < 1\}$. Find:

As before, draw a picture of the region. This will help you to set bounds for integration, and may provide a hint for part (d).

a. The joint density of (X, Y)

b. The marginal densities $f_X(x)$ and $f_Y(y)$

c. Are X and Y independent?

d. Find $\mathbb{E}[(X + Y)^2]$.

Problem 3

Minimum and maximum of two independent exponentials. Suppose S and T are i.i.d. Exponential (λ) random variables. Define $X = \min\{S, T\}$, $Y = \max\{S, T\}$, and $Z = Y - X$.

Consider $P(X \in dx, Y \in dy)$. What are the possible ways this could happen?

a. Find the joint density of X and Y. Are X, Y independent?

b. Find the joint density of X and Z. Are X, Z independent?

c. Find $\mathbb{E}(XY)$.

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