Stat 134: Section 18

Adam Lucas

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Problem 1

Let $X \sim \text{Geom}\,(p)$ on $\{1, 2, \ldots, \}$. Let $Y \sim \text{Uniform}\,\{0, 1, \ldots, X\}$ (that is, given $X = x$, Y is uniformly distributed from 0 to x).

1. Find $\mathbb{E}(Y|X = k)$;
2. Find $\mathbb{E}(Y)$.

Problem 2

Let $X_1, \ldots, X_n$ be independent Poisson random variables with parameters $\lambda_1, \ldots, \lambda_n$ respectively. Find the conditional distribution of X_1, given $X_1 + \cdots + X_n = N$ for $N \geq 1$. Is this a distribution that you know?

Problem 3: Exponential and Uniform spacing

Let $X_1, \ldots, X_n$ be independent Exponential random variables with parameter $\lambda = 1$, and let $X_{(1)}, \ldots, X_{(n)}$ be the order statistics.

a. Find the joint density of $Z = X_{(r)}, W = X_{(s)} - X_{(r)}$ for $1 \leq r < s \leq n$. Are Z, W independent?.

b. Prove that $e^{-X_{(n)}}, \ldots, e^{-X_{(1)}}$ has the same distribution with $(U_{(1)}, \ldots, U_{(n)})$ from $Unif[0,1]$, and conclude that $U_{(r)}/U_{(s)}, U_{(s)}$ are independent.