## Stat 134: Section 20

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## Conceptual Review

a. What is the computational formula for covariance?
b. If $X$ and $Y$ are independent, what is $\operatorname{Cov}(X, Y)$ ?
c. Use bilinearity of covariance to expand $\operatorname{Cov}(a X+Y, Y+Z)$, where $a$ is a constant.

## Problem 1

Let $X$ have uniform distribution on $\{-1,0,1\}$ and let $Y=X^{2}$. Are $X$ and $Y$ uncorrelated? Are $X$ and $Y$ independent? Explain carefully. Ex 6.4.5 in Pitman's Probability

## Problem 2

Let $A$ and $B$ be two possible results of a trial, not necessarily mutually exclusive. Let $N_{A}$ and $N_{B}$ be the number of times $A$ and $B$ respectively occur in $n$ i.i.d. copies of this trial. Show that if $N_{A}$ and $N_{B}$ are uncorrelated, then events $A$ and $B$ are independent.

What is this problem asking us to show? How does this connect to $\operatorname{Cov}\left(N_{A}, N_{B}\right) ?$

## Problem 3

Let $S$ and $T$ be random variables with variances $\sigma^{2}, \tau^{2}$ respectively. Suppose $\operatorname{Corr}(S, T)=\rho$. Find $\operatorname{Var}(3 S+2 T)$. (Hint: begin by finding $\operatorname{Cov}(S, T)$ based on the provided information.)

