Stat 134: Section 21

Adam Lucas

April 27th, 2023

Conceptual Review

a. What is Φ^{-1}?

b. Let X, Y be independent standard normal. Identify the distribution of:

    (i) X^2;

    (ii) $X^2 + Y^2$;

    (iii) $\sqrt{X^2 + Y^2}$

c. If X, Y are standard bivariate normal with correlation ρ, where $Y = \rho X + \sqrt{1 - \rho^2} Z$, what is the conditional distribution of Y given $X = x$? What about the conditional distribution of X given $Y = y$?

Problem 1

Heights and weights of a large group of people follow a bivariate normal distribution, with correlation 0.75. Of the people in the 90th percentile of weights, about what percentage are above the 90th percentile of heights?

Ex 6.5.3 in Pitman's Probability

Problem 2

Let X and Y be independent standard normal variables.

a. For a constant k, find $\mathbb{P}(X > kY)$.

b. If $U = \sqrt{3}X + Y$, and $V = X - \sqrt{3}Y$, find $P(U > kV)$.

c. Find $\mathbb{P}(U^2 + V^2 < 1)$.

d. Find the conditional distribution of X given $V = v$.

e. What is the joint distribution of U, V?

Ex 6.5.6 in Pitman's Probability

Problem 3

Show that for standard bivariate normal random variables with correlation ρ,

$$E[\max(X, Y)] = \sqrt{\frac{1 - \rho}{\pi}}.$$

Use that $2\max(X, Y) = X + Y + |X - Y|$.

Ex 6.5.11 in Pitman's Probability